

Concept Paper

Wave Age and Steepness Relation in Growing Wind Seas: A Case Study during Hurricane Matthew in 2016

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Received: March 12, 2023**Accepted:** June 15, 2023**Published:** June 20, 2023**Abstract**

For wind-wave interaction studies, from the literature, two criteria for the wind sea are used: one is the wave age and the other wave steepness. Analysis of pertinent datasets from a data buoy near the passage of Hurricane Matthew in 2016 indicates that $C_p/U_{10} = -15 H_s/L_p + 1.3$. Here C_p/U_{10} is the wave age and H_s/L_p is the wave steepness, where C_p is the phase speed of the peak wave, U_{10} is the wind speed at 10 m, H_s is the significant wave height, and L_p is the peak or dominant wave length. It is found that 87% of the variation between wave age and wave steepness can be explained by this relation. Application of this relation to estimate U_{10} and friction velocity from wave parameters are also presented for practical environmental and engineering use.

Keywords

Hurricane Matthew (2016); air-sea interaction; wind-wave interaction; wave age; wave steepness



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1. Introduction

In the realm of air-sea interaction, particularly in wind-wave interaction research, parameters of wave age and its steepness have been employed extensively (for literature reviews, see [1-3]). However, the relation between these two parameters has not yet been provided in the literature for practical use under hurricane conditions. The purpose of this concept paper is to present such a relation during the period of growing wind seas when Hurricane Matthew in 2016 (see www.nhc.noaa.gov) was near a data buoy owned and operated by the National Data Buoy Center (NDBC) (for the location and datasets of this buoy, see www.ndbc.noaa.gov). For detailed marine meteorological characteristics of Matthew, see [3].

2. Met-Ocean Measurements during Matthew

On October 2 and 3, 2016, Hurricane Matthew impacted the NDBC Buoy 42058 located in the deep central Caribbean Sea. During the growing wind seas, simultaneous meteorological and oceanographic (met-ocean) measurements of the wind speed at 5 m, U_5 , wind gust, U_{gust} , significant wave height, H_s , dominant wave period, T_p , and other parameters were made by the NDBC (see www.ndbc.noaa.gov) as provided in Table 1. Note that, during this 23-hour period, the wind speed increased from approximately 16 to 33 m s⁻¹, wind gust from 20 to 41 m s⁻¹, H_s from 4 to 10 m, and the barometric pressure dropped from 1002 to 958 hPa. This 44 hPa drop in 23 hour constitutes that Matthew was a meteorological bomb.

Table 1 Simultaneous measurements of meteorological and wave parameters during the period of growing wind seas at NDBC Buoy 42058 located in the Central Caribbean Sea near the passage of Hurricane Matthew in October 2016 (data source: www.ndbc.noaa.gov). Wave age and wave steepness parameters are computed (see text).

Day	Hour UTC	Min	U_5 m s ⁻¹	U_{gust} m s ⁻¹	H_s m	T_p sec	Baro hPa	H_s/L_p	C_p/U_{10}
2	6	50	16.1	20.4	4.16	10.81	1002	0.02	1.0
2	7	50	16.6	20.3	4.64	10	1001.2	0.03	0.9
2	8	50	15.9	20.2	5.7	10	1001	0.04	0.9
2	9	50	16.8	21.3	5.51	11.43	1001.2	0.03	1.0
2	10	50	18.2	22.4	6.14	11.43	1001.4	0.03	0.9
2	11	50	18	22.5	6.82	11.43	1001.1	0.03	0.9
2	12	50	18	23.8	7.22	10.81	1001.3	0.04	0.9
2	13	50	21	26.9	6.09	10.81	1000.9	0.03	0.7
2	14	50	20.5	24.9	7.07	11.43	1000.4	0.03	0.8
2	15	50	19.3	23.8	6.74	10.81	999.4	0.04	0.8
2	16	50	19.8	25.8	6.23	10	998.1	0.04	0.7
2	17	50	21.9	26.2	6.77	10	995.6	0.04	0.6
2	18	50	22.5	29.7	7.23	10	993.9	0.05	0.6
2	19	50	23.7	29.3	8.4	10	992.1	0.05	0.6
2	20	50	24.2	29.6	7.98	10.81	991	0.04	0.6

2	21	50	24.3	29.9	7.9	10	989.5	0.05	0.6
2	22	50	23.2	28.5	7.32	10	990.2	0.05	0.6
2	23	50	22.6	29.2	6.97	10	990.1	0.04	0.6
3	0	50	24	29	6.22	9.09	989.6	0.05	0.5
3	1	50	26.1	33.8	7.18	9.09	987.2	0.06	0.5
3	2	50	27.8	35	7.71	10	983.5	0.05	0.5
3	3	50	31.6	38.4	10.09	10	972.6	0.06	0.4
3	4	50	33	40.9	10.35	10.81	958.1	0.06	0.5

In order to minimize the effects of swell, conditions under the wind sea are investigated and acquired. According to [4], a wind sea is defined when

$$H_s/L_p \geq 0.020 \tag{1}$$

$$L_p = (g/2\pi)T_p^2 = 1.56T_p^2 \tag{2}$$

Here L_p is the dominant wave length in meters and g is the gravitational acceleration (= 9.8 m s⁻²). Note that the parameter H_s/L_p is called wave steepness, which is available from routine buoy measurements. On the other hand, a wind sea exists when the wave age, $C_p/U_{10} \leq 1.2$ (see, e.g., [5]). Here C_p is the phase speed of the peak wave so that

$$C_p = (g/2\pi)T_p = 1.56T_p \tag{3}$$

3. Adjusting the Wind Speed from 5 to 10 m

Because the wind speeds were recorded at 5-m instead of 10-m at Buoy 42058 during Matthew in 2016, one needs to adjust U_5 to U_{10} using the power-law wind profile (see, e.g., [6, 7]) that

$$U_{10}/U_5 = (10/5)^p \tag{4}$$

Here $p = (U_{gust}/U_5 - 1)/2 = (G - 1)/2$, where G is the gust factor and U_{gust} is the wind gust measured at the buoy [7].

Figure 1 shows that $G = 1.25$ so that $p = 0.125$. Substituting this p value into Eq. (4), we have,

$$U_{10} = 1.1U_5 \tag{5}$$

Using Eq. (5), we can now adjust the wind speed from 5 to 10 m.

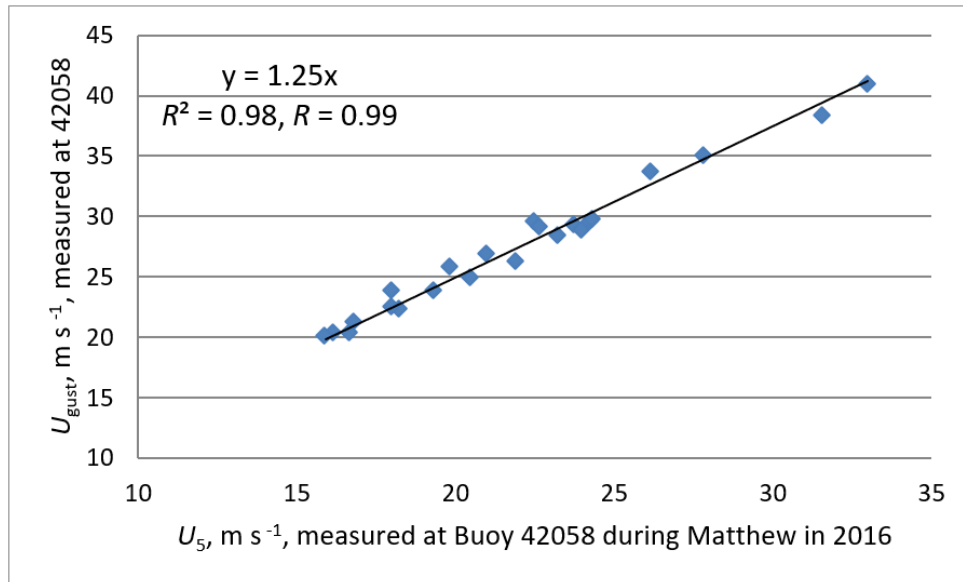


Figure 1 Gust factor, $G = U_{gust}/U_5$, measured at Buoy 42058 during Hurricane Matthew in 2016.

On the other hand, the logarithmic wind profile [6, 7] can also be employed, since

$$U_z = (U_*/k)\text{Ln}(Z/Z_o) \tag{6}$$

And from [8]

$$Z_o = 1200H_s(H_s/L_p)^{4.5} \tag{7}$$

Here U_z is the wind speed at height Z , U_* is the friction velocity, k ($= 0.4$) is the von Karman constant, and Z_o is the roughness length.

Since U_z is available from the NDBC buoy measurements and by eliminating (U_*/k) from Eq. (6), we have

$$U_{10}/U_z = \text{Ln}(10/Z_o)/\text{Ln}(Z/Z_o) \tag{8}$$

A comparison between the power-law wind profile based on Eq. (4) and the logarithmic wind profile from Eq. (8) is demonstrated as follows: During Hurricane Delta in 2020, NDBC Buoy 42002 (located in the western Gulf of Mexico) was impacted. Similar to Eq. (4), we have

$$U_{10}/U_{4.1} = (10/4.1)^p \tag{9}$$

Here $p = (U_{gust}/U_{4.1}-1)/2 = (1.32-1)/2 = 0.16$, where $G (=U_{gust}/U_{4.1})$ is the gust factor and U_{gust} is the wind gust measured at Buoy 42002 during Delta as provided in Figure 2 so that,

$$U_{10} = 1.15U_{4.1} \tag{10}$$

On the other hand, based on Eq. (8) and set $Z = 4.1\text{m}$, one gets

$$U_{10} = U_{4.1}\text{Ln}(10/Z_o)/\text{Ln}(4.1/Z_o) \tag{11}$$

Here Z_o is based on Eq. (7) by using the simultaneous wave measurements made at Buoy 42002 during Delta. Our result is presented in Figure 3. Since both slope and R^2 are near unity, we are confident to say that the power-law and log-law wind profiles are compatible. However, since the log-law requires both wind and wave datasets so that the wave age can be computed which may suffer the self correlation, we are using only the wind gust factor as employed in Figure 1 to adjust the wind speed from 5 to 10 m.

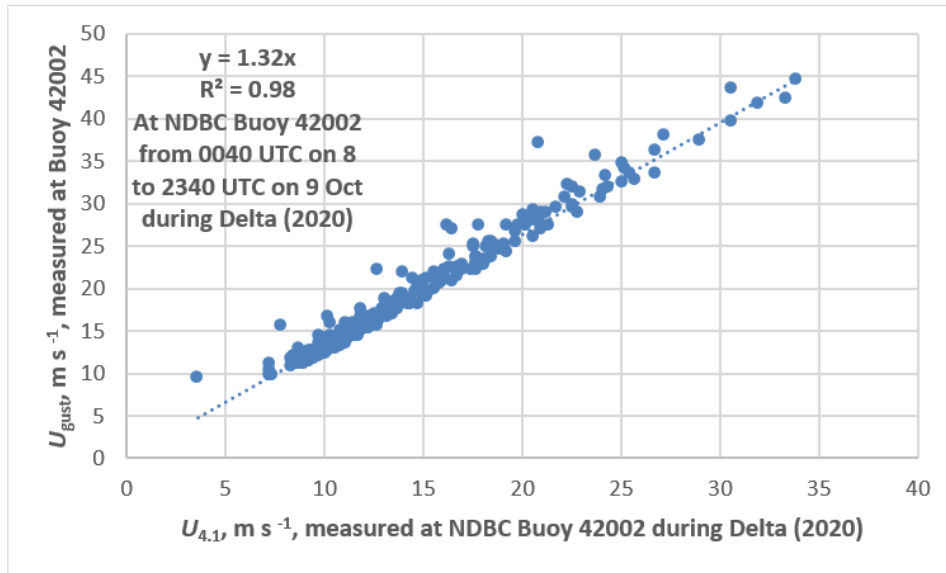


Figure 2 Measurements of the gust factor at NDBC Buoy 42002 during Hurricane Delta in 2020.

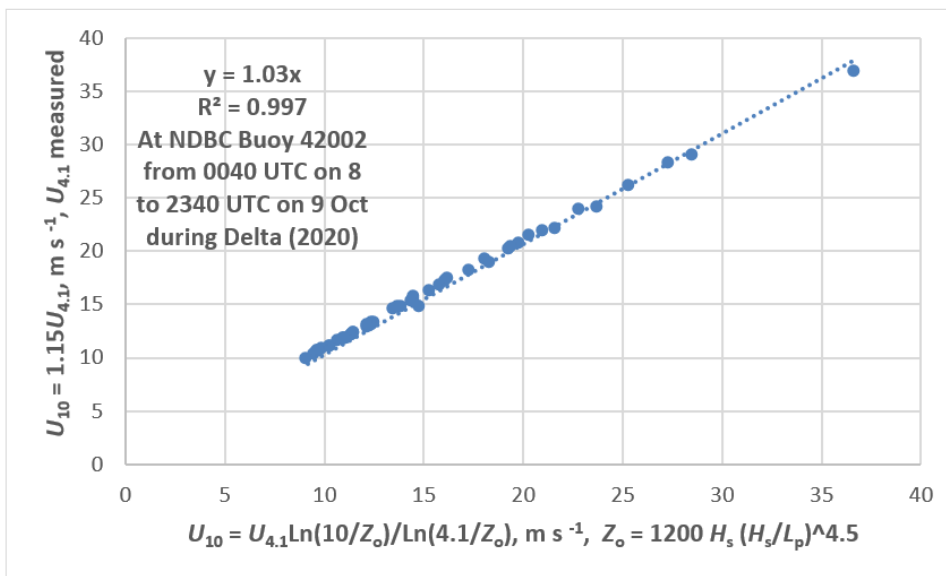


Figure 3 A comparison of U_{10} estimates based on the power-law (in vertical axis) and logarithmic wind profile law (in horizontal axis) at Buoy 42002 during Hurricane Delta in 2020.

4. Relation between Wave Age and Wave Steepness

On the basis of aforementioned methods and datasets from Table 1, our results are presented in Figure 4, which shows that

$$C_p/U_{10} = -15 H_s/L_p + 1.3 \tag{12}$$

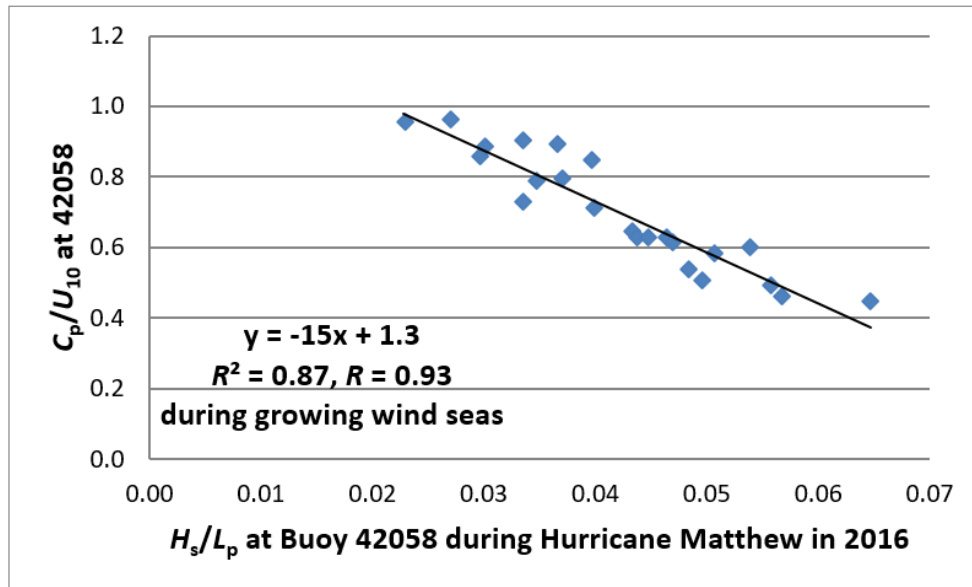


Figure 4 Relation between wave age (C_p/U_{10}) and wave steepness (H_s/L_p) based on the last 2 columns in Table 1.

As shown in Figure 4, the correlation coefficient ($R = 0.93$) is high, since the coefficient of determination, $R^2 = 0.87$, meaning that 87% of the variation between wave age and wave steepness can be explained by Eq. (12).

5. Applications

Rearranging Eq. (12), we have

$$U_{10} = C_p / (-15 H_s/L_p + 1.3) \tag{13}$$

Because NDBC Buoys 42003 and 42056 recorded all three parameters, i.e., the wind speed at 10 m, U_{10} , H_s , and T_p , during 4 hurricanes: Ivan (Sep. 13-16, 2004) and Katrina (Aug. 26-28, 2005) at 42003, Emily (July 17-18, 2005) and Wilma (Oct. 18-23, 2005) at 42056 (see www.ndbc.noaa.gov), these datasets are employed to compute U_{10} according to Eq. (13). Our results are presented in Figure 5. Since the slope is 94%, which is within the composite field accuracy of +/- 90% for the wind measurements on the NDBC buoys, according to the NDBC, and since the correlation coefficient is 95%, Eq. (13) may be used to estimate U_{10} from wave parameters during wind seas. Using 2 other independent datasets [9, 10] and a practical formula between U_{10} and H_s [11, 12], Eq. (13) is further verified as shown in Figure 6.

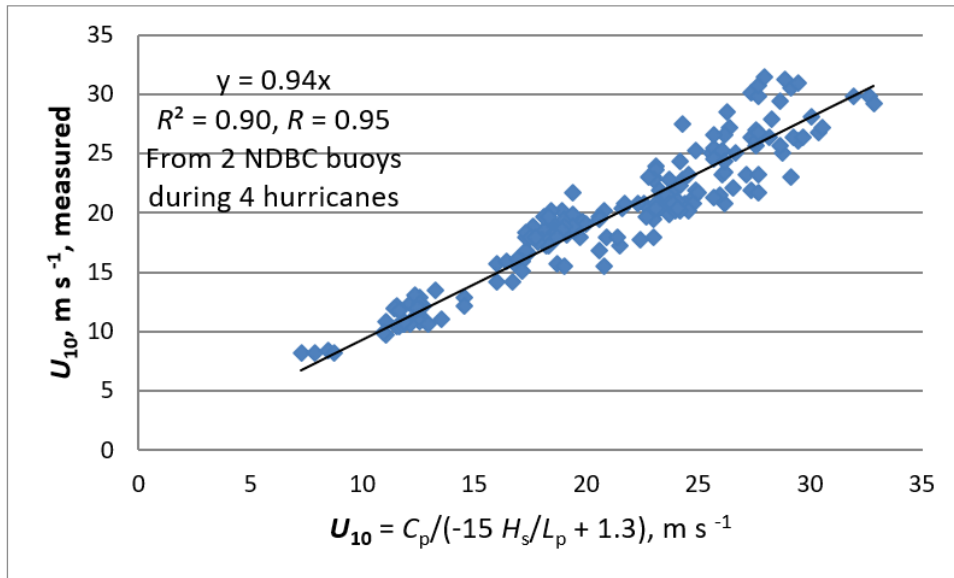


Figure 5 A comparison of Eq. (13) and measurements during 4 hurricanes (see text).

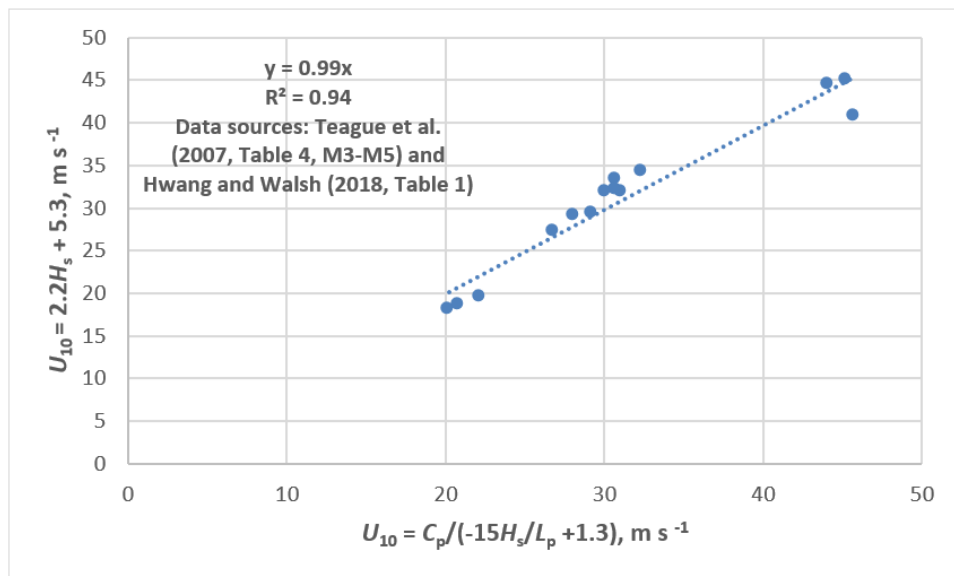


Figure 6 Further verification of Eq. (13) using 2 independent datasets.

In addition, using the estimated U_{10} and measured wave parameters, the shear or friction velocity, U_* , a basic parameter for air-sea interaction, can now be determined based on Eqs. (6, 7, and 13) so that

$$U_* = 0.4U_{10}/\ln(10/Z_0) \tag{14}$$

6. Conclusions

On the basis of aforementioned analysis and discussions, it is concluded that for the relation between wave age and wave steepness, Eq. (12) may be employed. Therefore, this research note reconciles the long-standing difference or question whether the wave age or the wave steepness is superior to use in wind-wave interaction investigations. However, since wave steepness is measured

routinely by the data buoys, it may be used to estimate the wave age using Eq. (12). Applications of this relation to estimate the wind speed at 10 m and friction velocity from wave parameters during the growing wind seas are also provided in Eqs. (13) and (14), respectively.

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Author Contributions

The author did all the research work of this study.

Competing Interests

The author has declared that no competing interests exist.

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